11.5 Videos Guide

11.5a

- The Alternating Series Test (statement and proof):
  If the alternating series
  $\sum_{n=1}^{\infty}(-1)^{n-1}b_n = b_1 - b_2 + b_3 - b_4 + \cdots$, where $b_n > 0$, satisfies
  (i) $b_{n+1} \leq b_n$ for all n
  (ii) $\lim_{n\to\infty} b_n = 0$
  then the series is convergent.

11.5b

Exercises:

- Test the series for convergence or divergence.
  - $\sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{\sqrt{n+1}}$
  - $\sum_{n=1}^{\infty}(-1)^{n+1}ne^{-n}$
  - $\sum_{n=1}^{\infty}(-1)^{n-1}\arctan n$

11.5c

Theorem (statement and proof):

- The Alternating Series Estimation Theorem: If $s = \sum_{n=1}^{\infty}(-1)^{n-1}b_n$ is a series that converges by the Alternating Series Test, then $|R_n| \leq b_{n+1}$.

Exercise:

- Use the Alternating Series Estimation Theorem to estimate the sum correct to four decimal places.

  $$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n}{8^n}$$

11.5d

Definitions: (absolute and conditional convergence)

- A series $\sum a_n$ is called absolutely convergent if the series of absolute values $\sum |a_n|$ is convergent.
- A series $\sum a_n$ is conditionally convergent if it is convergent but the series of absolute values $\sum |a_n|$ is divergent.

Theorem (statement and proof):

- If a series $\sum a_n$ is absolutely convergent, then it is convergent